ECONOMICS OF SOCIAL PROTECTION

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Outline of the lectures

INTRODUCTION

A. Design and sustainability

B. Performance of social protection

C. Social protection and private insurance

D. Tagging, transfers in kind and workfare

INTRODUCTION

0.1 Objectives

0.2 Size and structure across countries

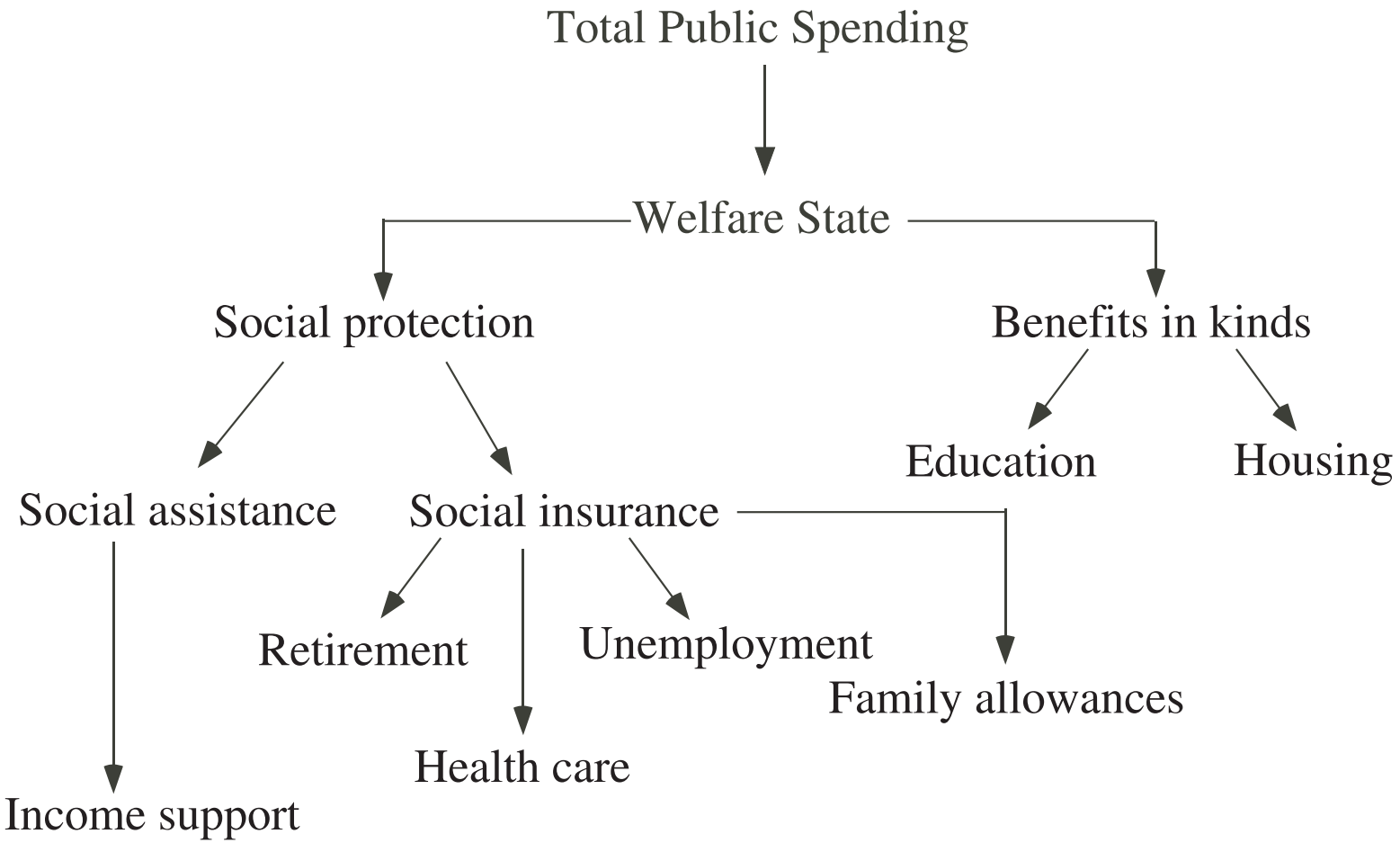
0.3 Taxonomy

0.4 Poverty and inequality

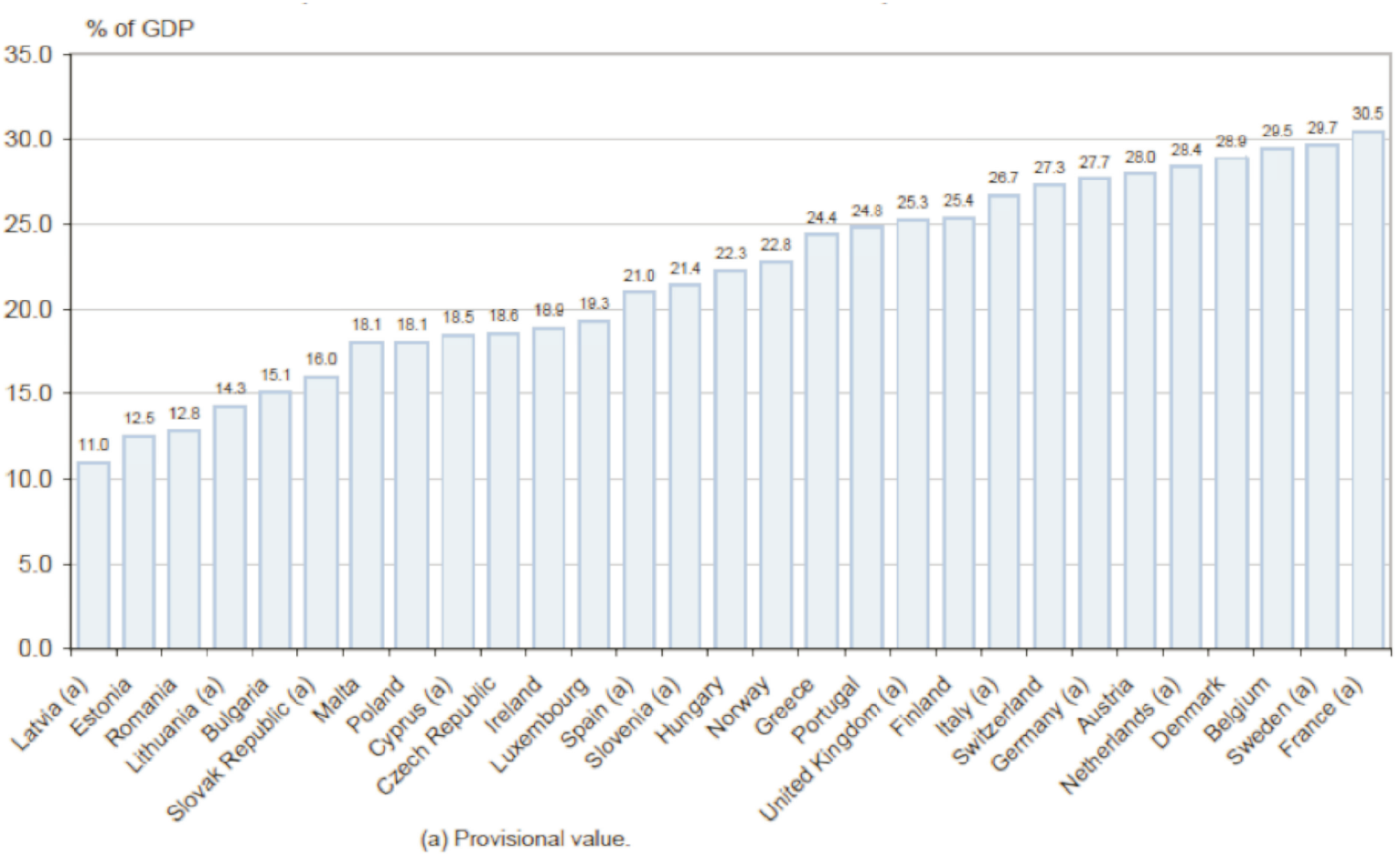
0.1. Objectives

- Protection against lifetime risks
  - Unemployment
  - Disability
  - Sickness
  - Early/late death
    - Retirement
    - Family
- Poverty alleviation
  - Relative versus absolute poverty
  - Temporary versus persistent poverty

Welfare state and social protection



Expenditure on Social Protection in the European Union, 2007



(a) Provisional value.

Source: Eurostat, Luxembourg 2010, Online Database, http://epp.eurostat.ec.europa.eu, accessed 18/02/2010.

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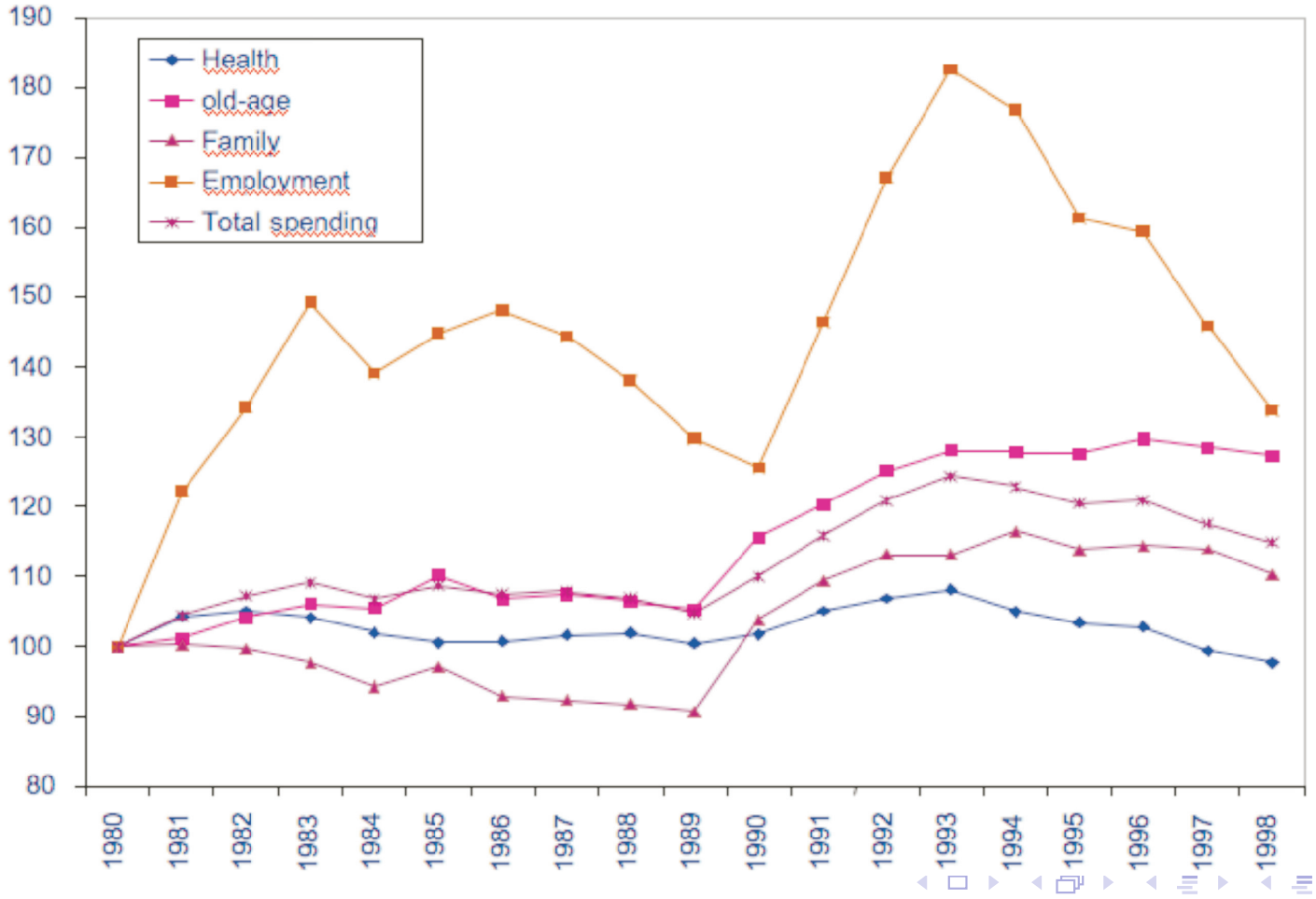
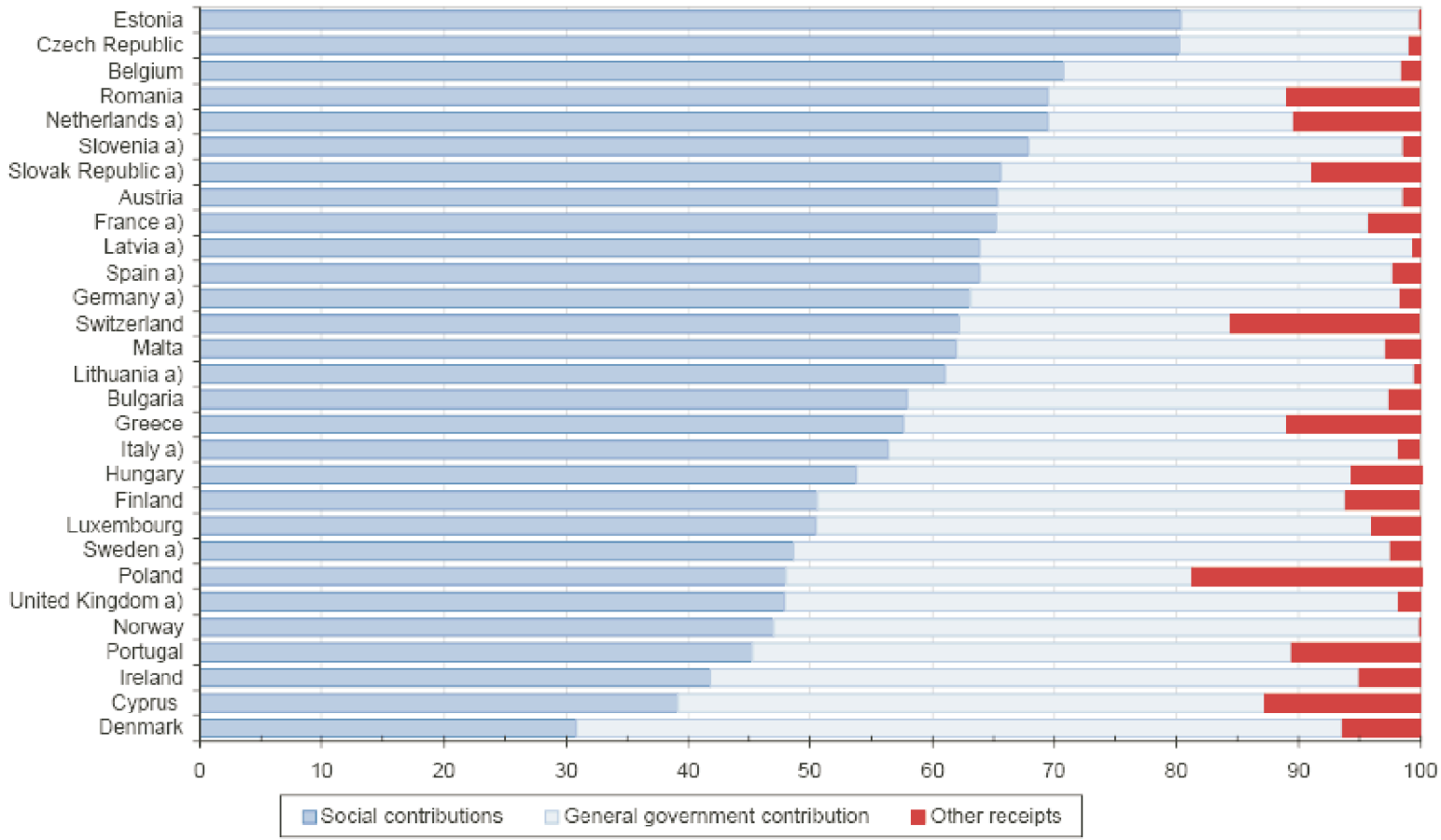
Social protection benefit by function, 2003

Country	Health	Old age	Family/housing	Labour market	Others	Total
Austria	30.0	51.6	11.6	4.9	1.9	100
Belgium	35.6	41.5	8.5	12.9	1.5	100
Denmark	37.4	28.5	15.2	15.3	3.6	100
Finland	36.9	35.8	13.4	11.8	2.1	100
France	32.8	42.7	12.9	10.2	1.4	100
Germany	37.7	44.1	7.7	8.6	1.9	100
Greece	28.9	55.7	10.6	2.3	2.5	100
Ireland	45.8	25.2	15.4	10.2	3.5	100
Italy	34.5	56.8	4.1	4.4	0.1	100
Luxembourg	40.0	38.8	17.1	3.1	1.0	100
Netherlands	44.9	32.4	6.9	12.9	2.9	100
Portugal	41.8	44.4	5.5	7.1	1.2	100
Spain	39.4	45.3	3.5	11.0	0.8	100
Sweden	43.6	34.0	12.3	8.0	2.2	100
United Kingdom	39.5	39.9	16.9	2.9	0.9	100
United States	49.7	41.4	2.6	3.1	3.2	100

Real growth of social spending by functions at the EC level (1980=100)



Sources of Funding for Social Protection, 2003



(a) Provisional Value.
Source: Eurostat, Luxembourg 2009, Online Database, http://epp.eurostat.ec.europa.eu, accessed 03/10/2009.

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Social protection systems can be classified according to:

- Generosity/redistribution
- Decommodification
- Activation
- Responsibility

Taxonomy of social transfers

Means tested		Categorical	
		Flats benefits (universalistic, Beveridgean)	Earnings-related benefits (social insurance, Bismarckian)
In cash	Welfare compensation		Unemployment compensation
In nature	Food stamps	Health services	—

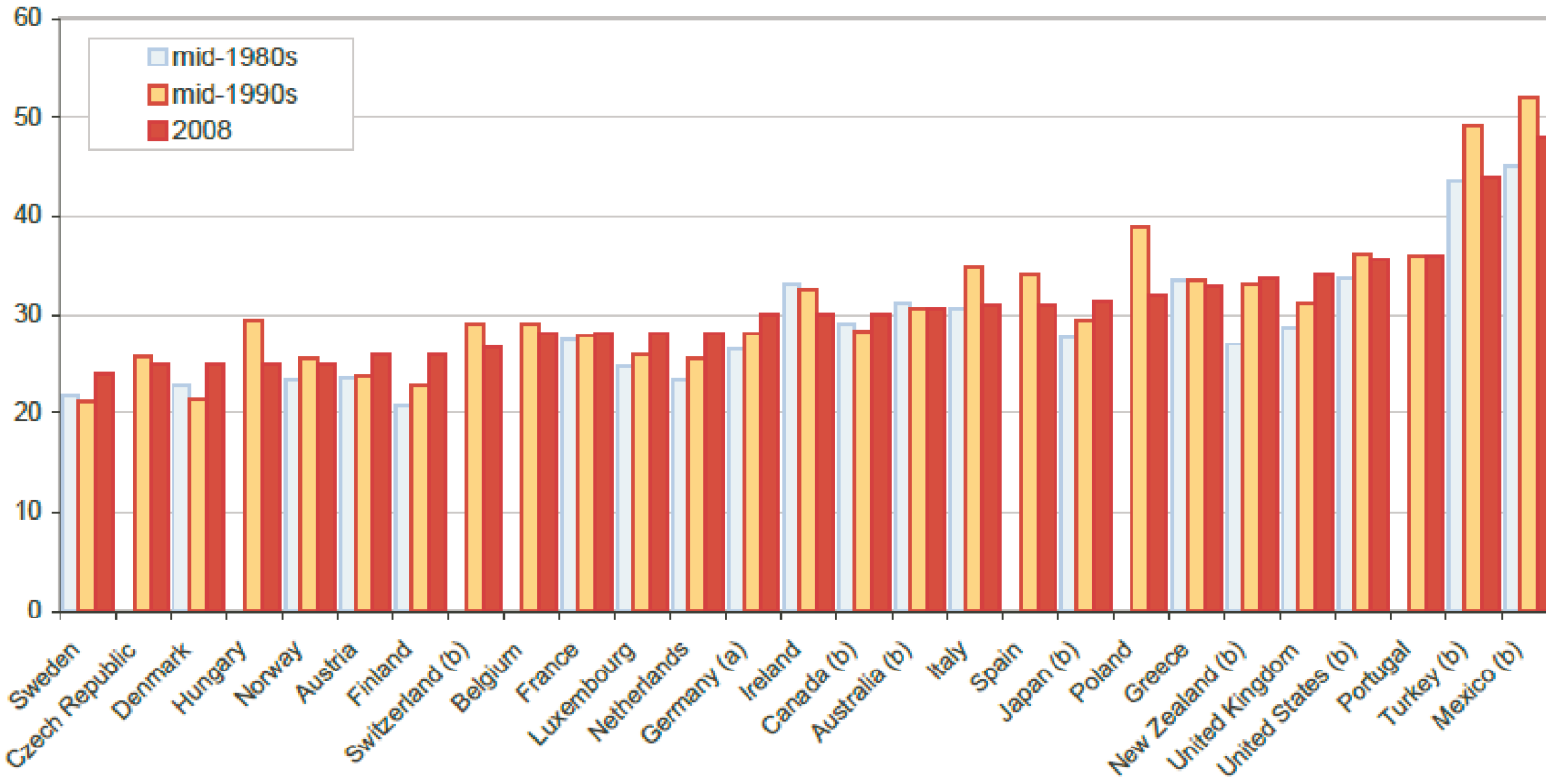
Indicators of the progressivity of pension benefit formulae – Gini coefficient for pension entitlements & progressivity index for OECD average

	Earnings distribution OECD average	
	Gini	Index
Austria	21.5	20.7
Belgium	9.5	64.8
Czech Republic	7.8	71.1
Denmark	2.3	91.7
Finland	25.3	6.7
France	14.6	46.4
Germany	20.9	22.9
Greece	26.0	4.3
Hungary	25.6	5.6
Ireland	0.0	100.0
Italy	26.1	4.0
Luxembourg	2.5	17.2
Netherlands	25.6	5.7
Poland	25.7	5.2
Portugal	18.7	31.1
Slovak Republic	26.3	13.0
Spain	23.6	13.0
Sweden	21.3	21.6
United Kingdom	8.3	69.6
Norway	14.8	45.3
Switzerland	15.2	44.1
Australia	6.8	74.8
Canada	3.7	86.5
Japan	14.2	47.8
New Zealand	0.0	100.0
United States	16.1	40.6

The first column shows the result for the Gini coefficient of gross pension benefits.
The second column shows the index of progressivity of the benefit formula.
In the flat-rate cases – Ireland and New Zealand – the index is 100.
Other countries with highly progressive pension systems are Australia, Canada, the Czech Republic, Denmark and the United Kingdom where the index is above two thirds. These countries have targeted of basic pensions that play a major role in retirement-income provision.
At the other end of the scale, Finland, Hungary, Italy, the Netherlands, Poland and the Slovak Republic have almost entirely proportional systems with very limited progressivity. The index is less than 10 in all these cases. This group includes two countries with notional accounts (Italy, Poland), which were deliberately designed to have a close link between contributions and benefits. Other countries lie between these two groups.

Source: OECD, Pensions at a Glance, Paris 2005, p. 82

Gini Coefficient of Inequality in the Distribution of Equivalised Household Disposable Income between mid-1980s and 2008



(a) Data for Germany in the mid-1980s refer to Western Germany only. – (b) Data for 2008 refer to 2000.

Source: OECD (2005), Society at a Glance, p. 55, and Eurostat, Online Database 2010, http://appsso.eurostat.ec.europa.eu/nui/show.do?dataset=ilc_sic2&lang=de (accessed 18/02/2010).

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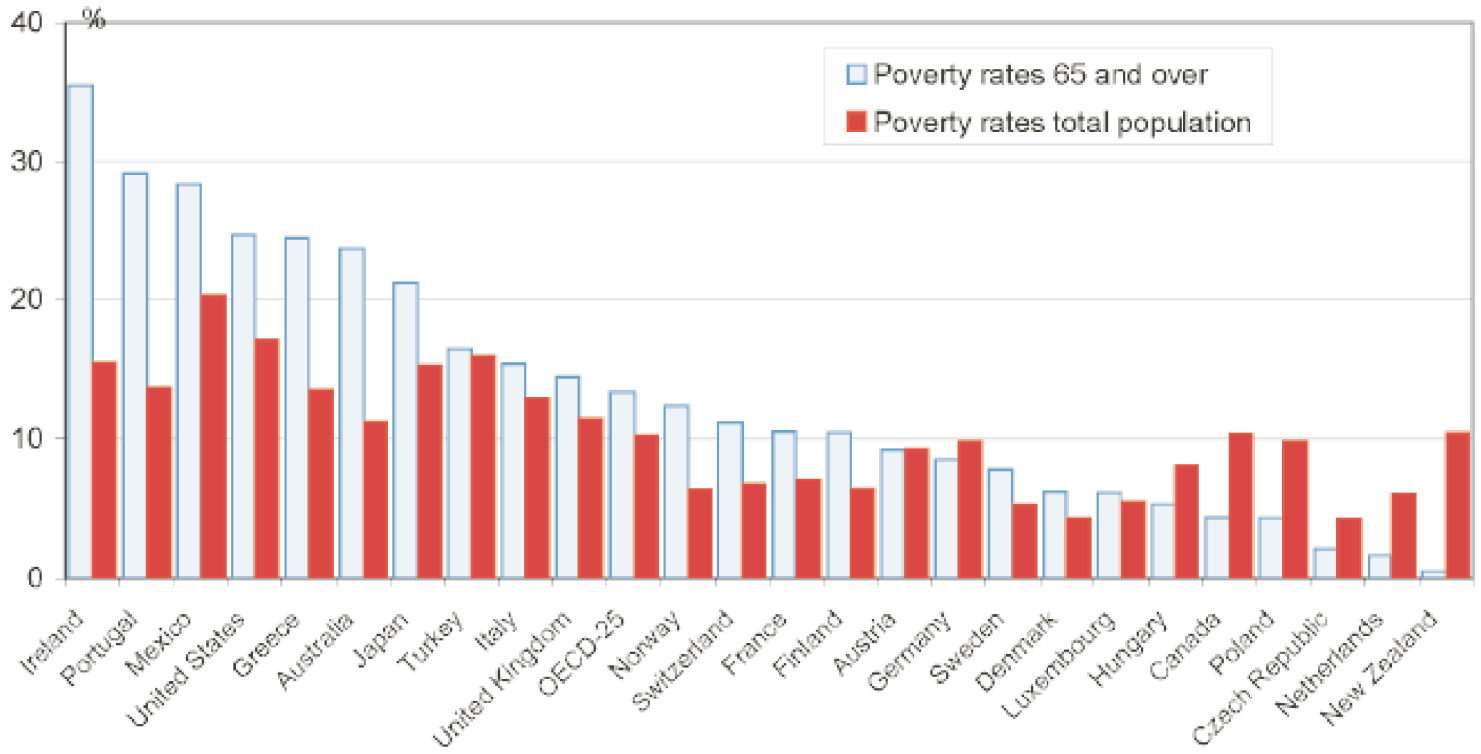
Poverty Rates[a)] among Older People and for the Total Population, 2000



(a) Poverty rates are measured as the proportion of individuals with equivalised disposable income less than 50% of the median income of the entire population.

Source: OECD (2005), Society at a Glance, Paris, 2005, p. 59.

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Poverty persistence: duration of the poverty spell over three years

	40% of median income	50% of median income			60% of median income
	Average of poverty rates over the three years	Average of poverty rates over the three years	Poor only once	Persistent poor	Average of poverty rates over the three years
Austria	3.6	7.1	6.7	2.9	13.0
Belgium	3.0	6.7	7.2	2.6	13.7
Denmark	2.4	5.7	7.3	1.7	11.0
Finland	2.9	6.8	5.9	2.8	12.9
France	3.6	7.9	7.6	2.8	14.9
Germany	3.3	6.1	5.6	2.3	11.0
Greece	9.1	14.6	9.7	7.1	21.4
Ireland	6.9	15.0	7.9	8.1	22.2
Italy	7.9	12.6	8.3	5.6	19.6
Luxembourg	1.7	5.9	4.4	2.6	13.0
Netherlands	2.9	5.2	5.7	1.3	9.6
Portugal	7.6	13.5	8.4	7.1	20.4
Spain	7.3	13.9	11.1	5.5	20.9
United Kingdom	6.5	11.9	9.4	5.1	19.2
Australia	6.2	14.0	10.5	6.5	20.8
Canada	6.9	11.8	7.4	6.6	18.7
United States	8.7	13.9	8.6	7.2	20.9

Note: Data refer to three years in the early 2000s. Relative income poverty is based on equivalised household disposable income. All measures are bases on the set of individuals present in each of the three-year period. Source: OECD, Society at a Glance: OECD Social Indicators – 2006 Edition, Paris 2007, p. 81.

A. DESIGN AND SUSTAINABILITY

A.1 Bismarck versus Beveridge

A.2 Optimal design

A.3 Political sustainability

A.4 Notional accounts. Pros and cons

A.1. Bismarck versus Beveridge

Example of pensions. Individuals differ in productivity w_i.

Utility:

$$u(c_i) + \beta u(d_i) = u(w_i(1-\tau) - s_i) + \beta u((1+r)s_i + p_i)$$

where

$$p_i = \tau(1+r)[\alpha w_i + (1-\alpha)\overline{w}]$$

α: Bismarckian parameter

$$U_i = u\big(w_i(1-\tau) - s_i\big) + \beta\, u\big((1+r)s_i + p_i + w_i z_i(1-\tau) - z_i^2/2\big)$$

where

$$\begin{aligned} p_i &= \tau\big[\big(\overline{wz} + (1+r)w_i\big)(1-\alpha) + \big(w_i z_i + (1+r)w_i\big)\alpha\big] \\ u'(c_i) &= \beta(1+r)u'(d_i) \\ z_i &= w_i\big(1-\tau(1-\alpha)\big) \end{aligned}$$

A.2. Optimal design

$\max \sum \nu_i n_i U_i$

where ν_i: social weight

n_i: proportion of type i's individuals

Key factors: liquidity constraints

tax distortions

individual characteristics

Solution: $\alpha \leq 0$

$1 > \tau > 0$

A.3. Political sustainability

Two stage choice

- choice of α (normative or positive)
- choice of τ (majority voting)

Solution: $1 > \alpha > 0$ τ increases with α

Empirical text:

τ: generosity

$1 - \alpha$: redistributive index

Correlation $(\alpha, \tau) = 0.74$

A.4. Notational accounts

Observation:

In most countries, 2/3 social spending concern lifetime redistribution

Conjecture:

Regardless of how much benefit one gets from paid contributions, these are perceived as taxes.

See above: regardless of α, people supply

$$\ell = w(1-\tau) \text{ and not } w\big(1-\tau(1-\alpha)\big)$$

Approach:

Divide social protection in two parts:

- Notional accounts for lifetime redistribution
  - Health
  - Education
  - Pension
- Redistribution programs for redistribution across households

Assessment: open questions

- Empirical issue that has not been solved?
- First step towards privatization ?
- Political sustainability?

B. THE PERFORMANCE OF SOCIAL PROTECTION

B.1 Performance approach

B.2 Performance and efficiency

B.3 Measuring and comparing

B.4 Convergence

B.1. Performance approach

- How to measure the performance of the WS?
- Social spending ?

In the EU15, share in GDP

20,6	in 1980
23,4	in 1990
24	in 2001

Not a good measure:

- other means (minimum wage, labor protection),
- mandatory and subsidized private schemes

We are interested by the outcome and not the means.

Performance evaluation

Relative to the objective of the WS

- poverty alleviation
- lifetime risk protection

B.2. Performance versus efficiency

- Analogy: difference between grading students and weighting their grades according to their merits
  - The best student is not necessarily the most deserving
- Performance: measured according to the way the objective are fulfilled
- Efficiency: can one improve performance with available resources?

Efficiency measurement: clear link between resources and performance

OK for railways, postal services, hospitals, ...

Questionable for education and health

Even more questionable for large aggregates: welfare state, social protection

B.3. Measuring and comparing

5 partial indicators: EU15, 1995-2008

- POV: poverty rate (60%)
- INE: interquintile ratios
- UNE: long term unemployment
- EDU: early school leavers
- EXP: life expectancy

Normalized from 0 to 1 (HDI)

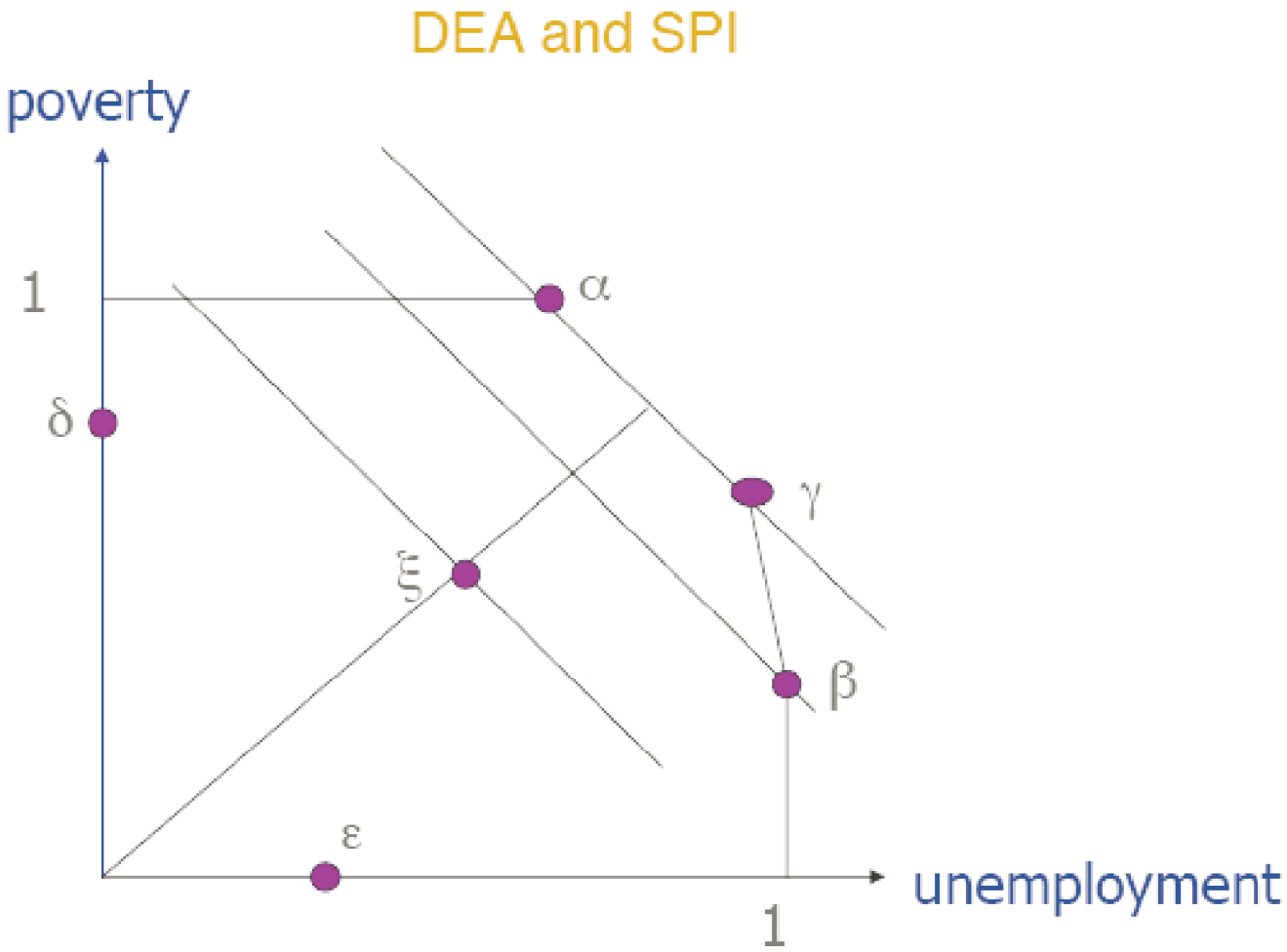
Other normalizations

Two ways to aggregate these partial indicators:

- unweighted sum (HDI)
  - sum of partial indicators (SPI)
- distance with respect to a best practice frontier

Best practice fontier:

- two types of methods
  - parametric
  - non parametric (DEA)


DEA and SPI
poverty
1
α
δ
γ
ξ
β
ε
unemployment
1

Performance scores and ranks, EU15 - 2008

	SPI		DEA	
	Scores	rank	Scores	rank
AUT	0.865	2	1.000	1
BEL	0.568	9	0.918	10
DEU	0.495	10	0.928	9
DNK	0.757	4	1.000	1
ESP	0.359	13	0.910	11
FIN	0.748	6	1.000	1
FRA	0.725	7	1.000	1
GBR	0.439	12	0.774	14
GRC	0.274	14	0.810	13
IRL	0.609	8	0.942	8
ITA	0.464	11	1.000	1
LUX	0.750	5	0.888	12
NLD	0.843	3	1.000	1
PRT	0.093	15	0.248	15
SWE	0.923	1	1.000	1
Mean	**0.594**		**0.895**	

DEA: with less indicators, less unitary scores

Correlation between SPI and DEA: 0.80

Other normalizations: same results

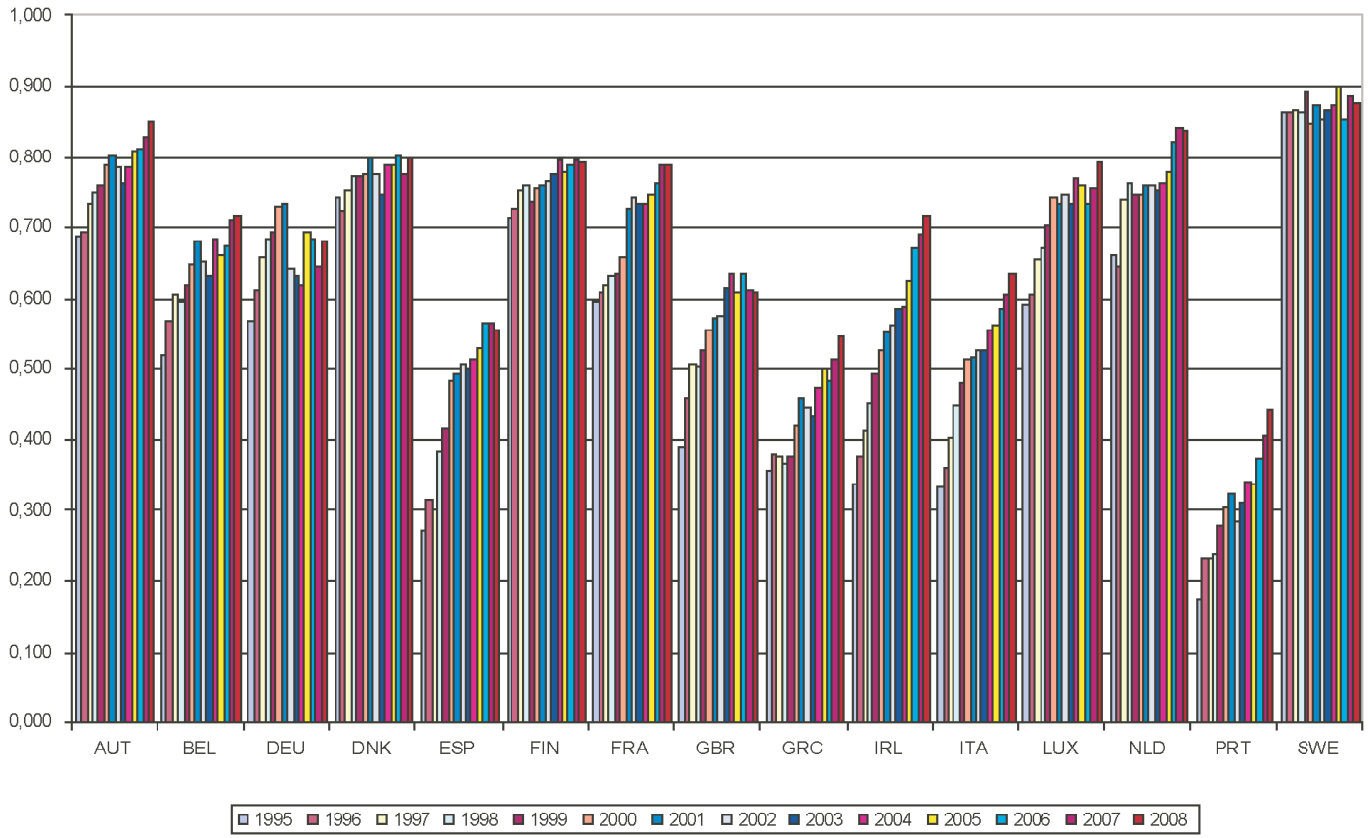
Performance scores and ranks, EU27-2008

	SPI		DEA		DEA-I	
	Scores	rank	Scores	rank	Scores	rank
AUT	0.885	2	1.000	1	0.770	20
BEL	0.728	12	0.921	19	0.691	27
BGR	0.392	25	0.737	27	0.788	17
CYP	0.813	8	1.000	1	1.000	1
CZE	0.852	6	1.000	1	1.000	1
DEU	0.680	13	0.931	17	0.702	25
DNK	0.857	5	1.000	1	0.712	23
ESP	0.551	21	0.973	14	0.980	10
EST	0.556	20	0.850	23	1.000	1
FIN	0.834	7	0.978	13	0.801	16
FRA	0.788	10	1.000	1	0.739	21
GBR	0.637	15	0.883	20	0.714	22
GRC	0.549	22	0.866	21	0.708	24
HUN	0.659	14	0.949	15	0.776	18
IRL	0.750	11	0.927	18	1.000	1
ITA	0.623	18	1.000	1	0.828	14
LTU	0.505	23	1.000	1	1.000	1
LUX	0.812	9	0.938	16	0.989	9
LVA	0.303	27	0.807	24	1.000	1
MLT	0.594	19	0.865	22	1.000	1
NLD	0.869	3	1.000	1	0.823	15
POL	0.633	16	1.000	1	0.949	12
PRT	0.425	24	0.778	25	0.696	26
ROM	0.354	26	0.755	26	0.858	13
SVK	0.632	17	1.000	1	1.000	1
SVN	0.867	4	1.000	1	0.954	11
SWE	0.906	1	1.000	1	0.771	19
Mean	**0.669**		**0.932**		**0.861**	

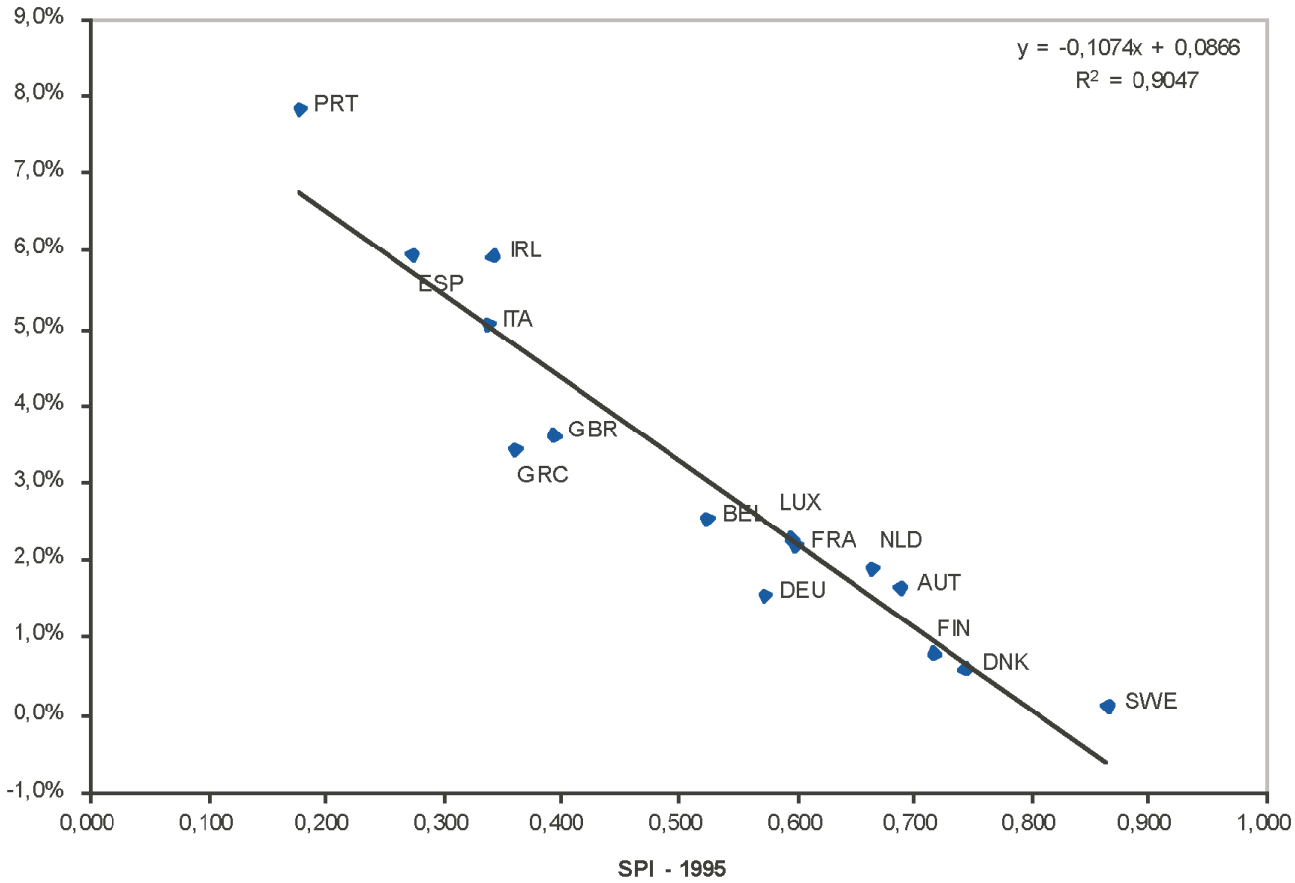
B.4. Evolution and convergence

- Hypothesis: economic integration and OMC lead to decreasing spending and redistribution and to convergence of performance
- How to measure? Spending or outcomes?
- Average indicator increases everywhere and converges

Average indicator 1995-2008

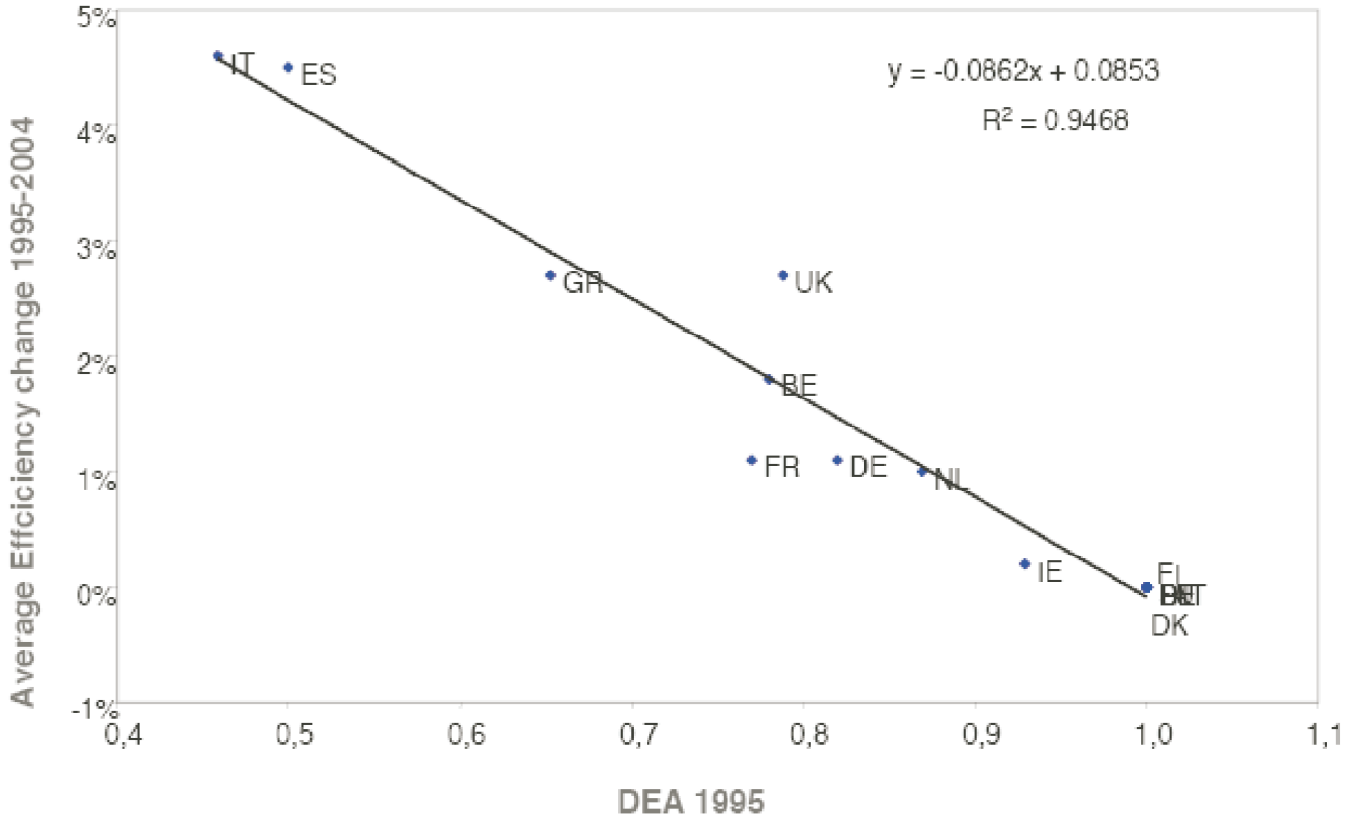


Convergence of SPI, EU15



- Variation in DEA performance measures
- Decomposition:
  - shift of the best practice frontier
  - variation in the distance w.r.t. the best practice frontier
- Malmquist measure and decomposition

Convergence of DEA according to "technical efficiency" change



- Performance (and efficiency when possible) evaluation is a **must**:
  - to check convergence and social dumping
  - to guide reforms
  - to foster yardstick competition (OMC)
- Possible disagreement on data and performance indicators
  - not a reason to discard them
- Room for improvement
- Final word: no sign of social dumping

C. SOCIAL PROTECTION AND PRIVATE INSURANCE

C.1 Insurance market

C.2 Social insurance and redistribution

C.3 Annuity market and long term care

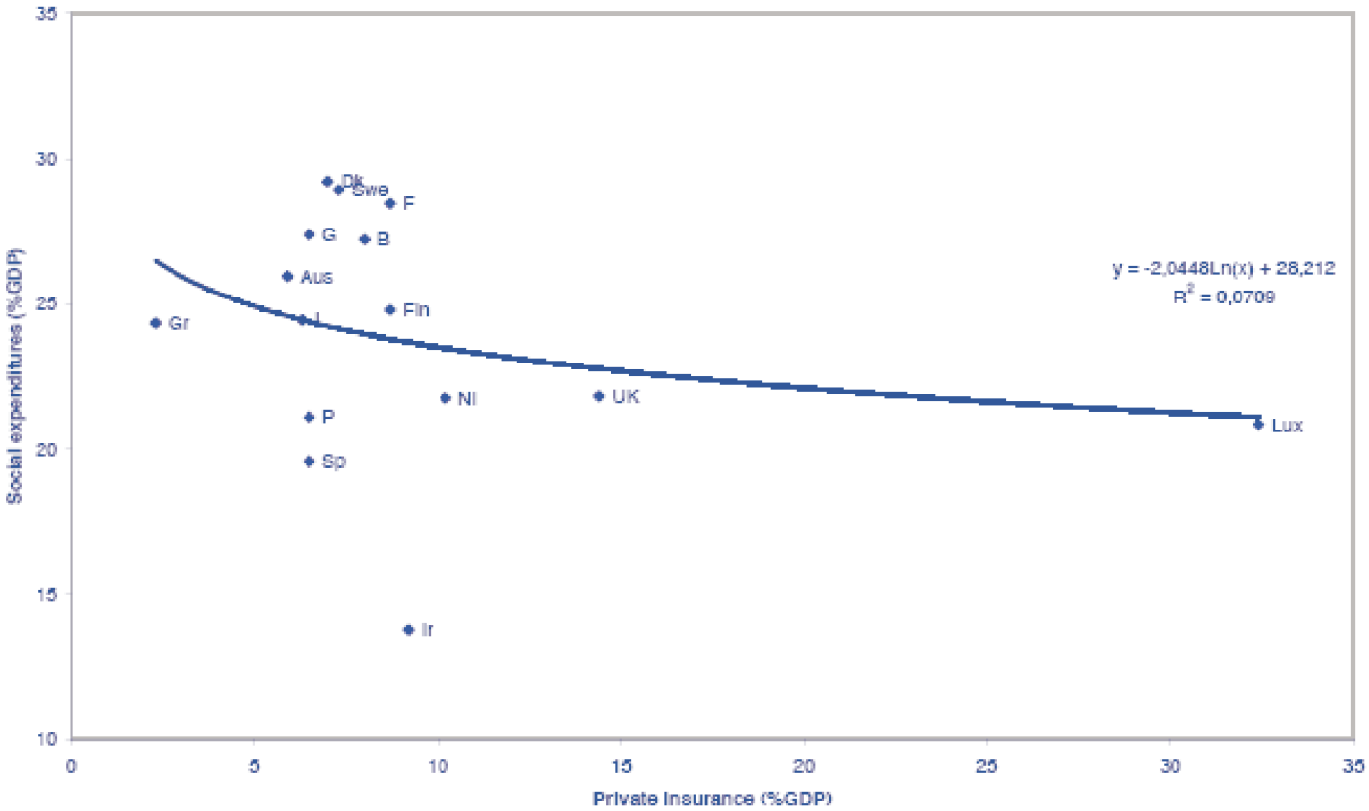
C.4 Health insurance and moral hazard

C.1. Insurance market

Social insurance and redistribution

Argument	Advantage of social versus private insurance
Large risk	Nil when reinsurance is possible
Intergenerational smoothing	High
Moral hazard	Negative
Adverse selection	Nil if insurance is made mandatory
Administrative cost	Noticeable particularly in the field of health care
Redistribution	High
Financing	Negative because of tax competition
Commitment	Negative
Single provision	High in the field of health care

Social spending and private insurance as share of GDP, 2001



C.2. Social insurance and redistribution

Individuals are characterized by their productivity w_i, and their probability of incurring a monetary loss D, p_i. There are two types of insurance: private (actuarially fair) and social (redistributive). Given risk aversion and actuarially fair) and social (redistributive). Given risk aversion and actuarial fairness, there will be full insurance.

Disposable income in both states of nature is:

$$x_i = w_i \ell_i - T_i(\alpha \overline{p} + (1 - \alpha) p_i) D$$

where T_i can be either non linear or linear and α is the coverage rate.

Linear case: $T_i = t\, w_i \ell_i - a$

Problem of the social planner:

$$\mathfrak{L} = \sum n^i \Big[[u(w_i \ell_i (1-t) + a - (\alpha \overline{p} + (1-\alpha) p_i) D - \nu(\ell_i)] - \mu (a - \sum t\, w_i \ell_i) \Big]$$

where $\ell_i = \ell\big((1-t) w_i\big)$.

$$\frac{\partial \mathfrak{L}}{\partial \alpha} = \sum n^i u'(x_i)(p_i - \overline{p}) > 0$$

if p_i and w_i are negatively correlated.

$$\begin{aligned} \frac{\partial \mathfrak{L}}{\partial t} &= -\sum n^i u'(x_i) w_i \ell_i + \mu \sum \left(w_i \ell_i + t\, w_i \frac{\partial \ell_i}{\partial t} \right) \\ &= -\text{cov}(u'(x_i), w_i \ell_i) + \mu\, t\, w_i \frac{\partial \ell_i}{\partial t} = 0 \end{aligned}$$

Extensions

	α
Loading cost	$+$
Moral hazard	$-$
Adverse selection	$+$

C.3. Annuity market

The "forgotten half" of retirement security is carefully planning the "payout phase".

Financial planning would be easy if we knew with certainty how long we each would live. But length of life is highly uncertain.

Uncertainty forces one to trade-off two risks:

- if consume too aggressively, you will "run out of resources" before you die
- if consume too frugally, you lower your standard of living

Life annuities as the solution

Individual trades a stock of wealth for a flow of income that lasts as long as individual lives.
An annuity can provide a higher level of sustainable income that can be achieved from a non-annuitized asset.

Is it magic?

No, "there is no free lunch".
Annuities pay a higher return when alive in exchange for giving up right to wealth upon death

Advantages of annuities

- higher return while living
- guaranteed income as long as you will live.

Disadvantages of annuities

- cannot bequeath the money (no inheritances)
- if annuity market are poorly developed, and thus there are concerns about:
  - pricing
  - liquidity

Economic theory of life annuities

Yaari (1965): under certain conditions, individuals should convert 100% of their wealth to annuities:

- no bequest motives
- actuarially fair annuities
- Von Neumann-Morgenstern expected utility
- exponential discounting
- utility of consumption is additively separable over time
- no uncertainty other than date of death

Illustration

$$u(c) + \pi\,\beta\,u(d).$$

We assume $\beta(1+r) = 1$ and define the rate of return on the annuity: $\rho = ((1+r)/\pi) - 1$.

$$\begin{aligned} \max \quad & u(w-s) + \pi\,\beta\,u\left(s\frac{1+r}{\pi}\right) \\ & \therefore u'(c) = u'(d). \end{aligned}$$

If there is no annuity:

$$u'(c) = u'(d)\pi.$$

Take $u = \ln$, $\pi = 1/2$, $r = 0$.

With annuity, $c = d = (2/3)w$.

Without annuity, $c = (2/3)w$; $d = (1/3)w$.

Davidoff, Brown and Diamond (2005) show that, with complete markets, sufficient conditions for optimality of full annuitization are:

- no bequest motives
- annuity return to survivors $>$ conventional asset return.

If markets are complete, the optimality of full annuitization survives the extension of the problem to many time periods and many states.

With incomplete markets, result can fail:

- if there is a severe mismatch between the desired consumption path and the annuity income stream, full annuitization sub-optimal
- example: some forms of expenditure shocks (i.e., medical shocks early in life).

The annuity puzzle

Economic theory says annuities are quite valuable and that retirees ought to hold most of their portfolio in this form. Empirical evidence is that most individuals do not voluntarily annuitize their resources.

Why ?

- bequests
- high prices (adverse selection or administrative costs)
- families as substitutes
- high discount rates
- uninsured medical expenditure shocks

Long term care

Huge problem linked to:

- Aging (share of 80+)
- Evolving family solidarity (20% without family support)
- Market failure

Current situation:

- Most assistance comes from the family
- Few schemes of social insurance (Germany)
- Narrow markets: except in the US (6 mo) and France (3 mo)

Definition: loss of autonomy

- Inability to perform basic daily activities (bathing, dressing, eating, continence)
- Need for assistance (different illness, disability, handicap, ...)
- Katz scale

Three major risks

- Escalating costs
- Adverse selection (risk of dependency, longevity)
- Moral hazard

Two types of contracts

- French, lump-sum reimbursement
- American, cost reimbursement

Long term care puzzle

- Excessive costs (loading factors and adverse selection)
- Social assistance acting as Good Samaritan
- Trust into family solidarity
- Unattractive rule of reimbursement (lump sum)
- Myopia or ignorance
- Denial of heavy dependence

C.4. Health insurance and moral hazard

Unobserved action

Undertaken before or after the health risk has materialised: ex post or ex ante moral hazard

Illustration

2 states of the world

- m with probability p and loss L
- b $1-p$

e: monetary effort
$h(e)$: loss reduction
r: coinsurrance
π: actuarially fair premium
y: initial income

Utility

$$U = p\,u[y - L + h(e) - e(1-r) - \pi] + (1-p)u[y - \pi]$$

Perfect observability: insurer chooses e, r and $\pi = p\,r\,e$

$$\frac{\partial U}{\partial r} = p\,u'(c_m)e(1-p) - (1-p)u'(c_b)pe = 0$$

$$\frac{\partial U}{\partial e} = p\,u'(c_m)[h'(e) - (1-r) - pr] - (1-p)u'(c_b)pr = 0$$

$$\therefore u'(c_m) = u'(c_b)$$

$$h'(e) = 1$$

$$r = 0$$

Asymmetric information

Indirect control of e:

$$\begin{aligned} 1-r &= h'(e) \\ e &= e(r), e'(r) > 0 \end{aligned}$$

Insurer maximizes U with respect to r and π s.t. $e = e(r)$ and $\pi = p\,r\,e(r)$.

$$\frac{\partial U}{\partial \pi} = -p\,u'(c_m) - (1-p)u'(c_b) + \mu = 0$$

$$\frac{\partial U}{\partial r} = p\,u'(c_m) - \mu\,p(e + r\,e'(r)) = 0$$

where μ is the Lagrange multiplier

$$r = \frac{e(1-p)[u'(c_m) - u'(c_b)]}{[p\,u'(c_m) + (1-p)u'(c_b)]e'(r)}$$

D. TAGGING, TRANSFERS IN KIND AND WORKFARE

D.1 Optimal redistribution

D.2 Transfer in kind

D3 Tagging

D.4 Workfare

D.1. Optimal redistribution

Ideally: lump-sum (non distortionary) redistribution

Second-best: unavoidable distortion

Optimal income tax: nonlinear/linear

$\therefore$ No need to use other instruments
(tax on luxury goods; subsidy or necessity, health, education)

Yes but: - non separable utility function
- heterogenity in more than one characteristic
- tax evasion

Cash transfer dominates in kind transfer.

Superiority of a lump-sum cash tax over a distortionary tax or a transfer in kind

Take an individual with utility $u(c,\ell) = c - \ell^2/2$ where $c = w\ell$ and $w = 10$.

In the LF, he chooses $\ell = 10$ which implies $c = 100$ and $u(c,\ell) = 50$.

Suppose that the government wants to give him a flat benefit of 16 to be financed either with a lump sum tax 16 or a proportional tax τ such that $\tau\, w\, \ell = 16$.

With the lump sum tax:

$$u = 10\ell - 16 + 16 - \ell^2/2 = 50.$$

Nothing changes.

With the flat tax, $u = 10(1-\tau)\ell + 16 - \ell^2/2$ where

$$\tau\, w\, \ell = \tau(1-\tau)w^2 = 16$$
$$\tau(1-\tau) = 0.16 \text{ or } \tau = 0.2$$
$$u = 32 + 16 = 48$$

Suppose now that the government wants to make a transfer in kind equal to 120 to be financed by a lump sum tax. To produce 120, our individual has to work not 10 but 12 hours. His utility is now

$$120 - 72 = 48$$

D.2. Transfer in kind

Transfer in kind

2 individuals

$$u_A = y_A$$

$$u_D = y_D - e^{1-z} = y_D - 1 \qquad z = 1$$

$$= y_D - 2.7 \qquad z = 0$$

$$y_A + y_D + z = 6$$

Objective: $u_A = u_D$

Perfect observation

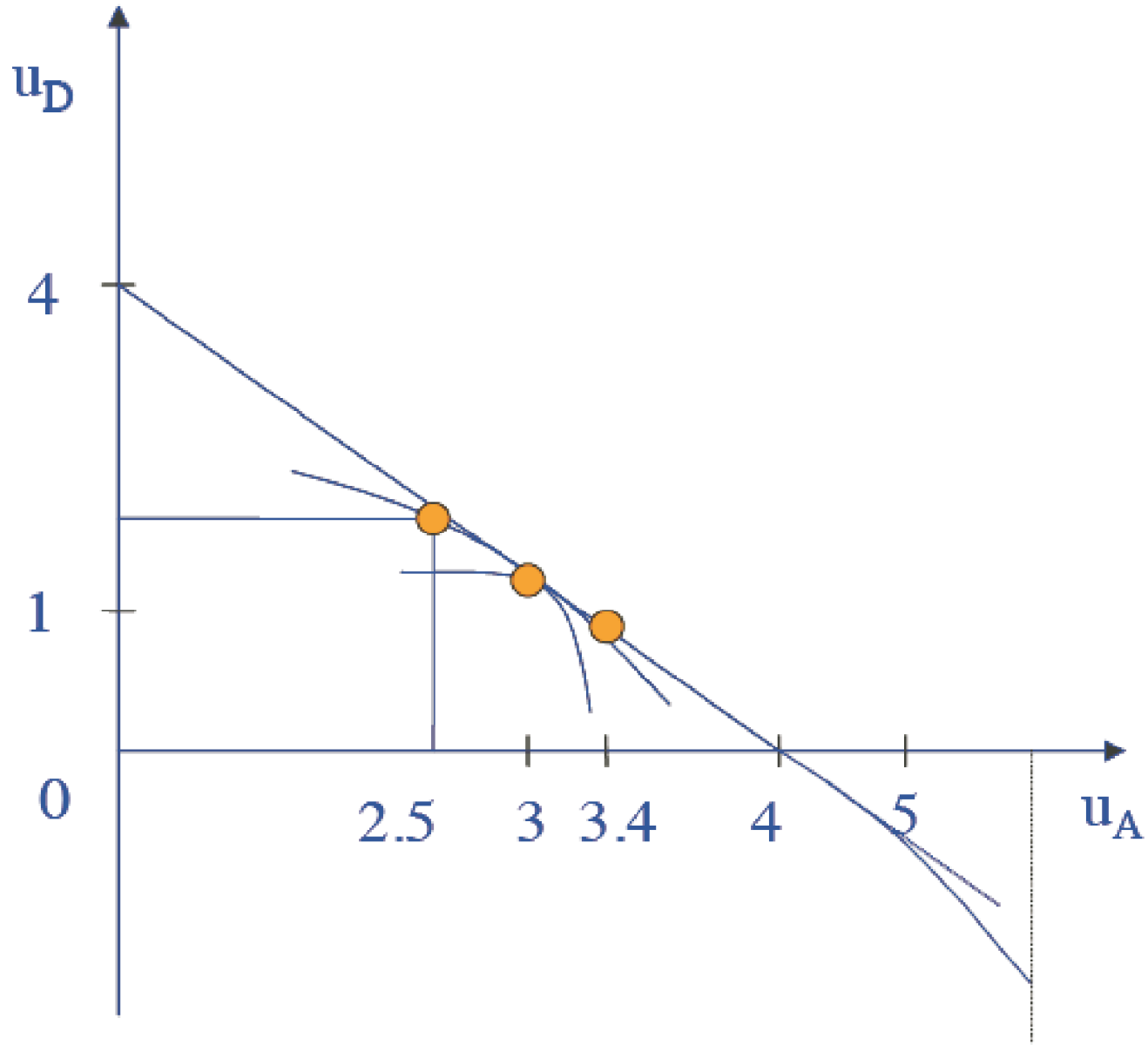
$$u_A = u_D = 2$$
$$y_D = 3, z = 1$$
$$y_A = 2$$

Types are not observed and z is supplied freely

$$y_A = y_D = 5/2$$
$$u_A = 2.5$$
$$u_B = 2.5 - 1 = 1.5$$

Only cash transfers:

$$y_A = y_D = 3$$
$$u_A = 3$$
$$u_D = 1$$


u_D
4
1
0
2.5
3
3.4
4
5
u_A

D.3. Disability and tagging

Issues

Stigmatisation, taking-up, horizontal equity, political support, errors

Model of income taxation

Two types of activities:

- a harsh one demanding skill: 2
- an easy and unskilled one: 1

2 types A and D

Only a fraction γ of D is tagged

$$\begin{aligned} u_A &= \log c_A - \varepsilon,\ w_A = 10 \\ u_D &= \log c_D, w_D = 0, 10^{\varepsilon} = 1.5, \varepsilon = 0.176 \end{aligned}$$

Problem of the central planner

$$\begin{aligned} \max \quad & \log c_A - \varepsilon + \gamma \log c_T + (1-\gamma) \log c_D \\ & -\mu[c_A + \gamma c_T + (1-\gamma)c_D - w_A] \\ & +\lambda[\log c_A - \varepsilon - \log c_D] \end{aligned}$$

$$\frac{1}{c_A} - \mu + \lambda c_A; \frac{\gamma}{c_T} - \gamma\mu = 0;$$

$$\frac{1-\gamma}{c_D} - (1-\gamma)\mu - \frac{\gamma}{c_D} = 0$$

- First-best

$$c_A = c_D = c_T = 5$$

- No tag ($\gamma = 0$)

$$\mu = 0.1$$
$$\lambda = 0.2$$
$$c_A = 6, \quad u_A = u_D$$
$$c_D = 4$$

- Tag

$$c_A = \frac{1+\lambda}{\mu}, c_T = \frac{1}{\mu}, c_D = \frac{1-\gamma-\lambda}{(1-\gamma)\mu}$$
$$\mu = 0.2, \quad c_T = 5$$
$$\lambda = \frac{0.5(1-\gamma)}{2.5-\gamma} = 0.2 \text{ if } \gamma = 0$$
$$= 0 \text{ if } \gamma = 1$$

D.4. Workfare

$$w_A > w_D$$

$$u_i = y_i - \ell_i^2/2 = w_i \ell_i - \ell_i^2/2 = \frac{w_i^2}{2}$$

Objectives of the government: make sure that everyone earns

$$z > w_D \ell_D = w_D^2$$

- FB: Transfer $b_D = z - w_D^2$
  Cost of the scheme:γb_D
- SB with transfer

$$b_D = b_A = b = z - w_D^2$$

  Cost of the scheme b_D
- SB with workfare c

$$\text{Now } b = z - w_D(\ell_D - c) = z - w_D^2 + w_D\, c$$

Find c and b that induce A to work normally

$$\begin{aligned}\frac{w_A^2}{2} &= w_A(\ell_A - c) - \frac{\ell_A^2}{2} + b \\ &= \frac{w_A^2}{2} - cw_A + b \\ & cw_A + z + cw_D - w_D^2 = 0\end{aligned}$$

$$c^* = \frac{z - w_D^2}{w_A - w_D}$$

$z - w_D^2 = c^*(w_A - w_D)$ is the cost of the transfer scheme. The cost of workfare is $\gamma[z - w_D^2 + w_D c^*] = \gamma w_A c^*$
Workfare dominates pure transfers if

$$\begin{aligned}(w_A - w_D)c^* &> \gamma\, w_A c^* \\ (1-\gamma)w_A &> w_D\end{aligned}$$